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Robust adaptive boundary control of a flexible marine riser with vessel dynamics☆

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ABSTRACT

In this paper, robust adaptive boundary control for a flexible marine riser with vessel dynamics is developed to suppress the riser's vibration. To provide an accurate and concise representation of the riser's dynamic behavior, the flexible marine riser with vessel dynamics is described by a distributed parameter system with a partial differential equation (PDE) and four ordinary differential equations (ODEs). Boundary control is proposed at the top boundary of the riser based on Lyapunov's direct method to regulate the riser's vibration. Adaptive control is designed when the system parametric uncertainty exists. With the proposed robust adaptive boundary control, uniform boundedness under the ocean current disturbance can be achieved. The proposed control is implementable with actual instrumentation since all the required signals in the control can be measured by sensors or calculated by a backward difference algorithm. The state of the system is proven to converge to a small neighborhood of zero by appropriately choosing design parameters. Simulations are provided to illustrate the applicability and effectiveness of the proposed control.

1. Introduction

With the increased focus on offshore oil and gas development in deeper and harsher environments, vibration control of the flexible marine risers has gained increasing attention. The marine riser is used as a fluid-conveyed curved pipe drilling crude oil, natural gas, hydrocarbons, petroleum materials, mud, and other undersea economic resources, and then transporting those resources in the ocean floor to the production vessel or platform on the ocean surface (Kaewunruen, Chiravatchradj, & Chucheepsakul, 2005). A drilling riser is used for drilling pipe protection and transportation of the drilling mud, while a production riser is a pipe used for oil transportation (How, Ge, & Choo, 2009). Vibration and deformation of the riser due to the ocean current disturbance and tension exerted at the top can produce premature fatigue problems, which require inspections and costly repairs. Recent advance in computer and electronics technology have allowed the development of complex electromechanical control systems to suppress the riser's vibration.

For the purpose of dynamic analysis, the riser is modeled as an Euler–Bernoulli beam structure with PDEs since the diameter-to-length of the riser is small. Based on the distributed parameter model, various kinds of control methods integrating computer software and hardware with sensors and actuators have been investigated to suppress the riser's vibration. In Do and Pan (2008), boundary control for the flexible marine riser with actuator dynamics is designed based on Lyapunov's direct method and the backstepping technique. In How et al. (2009), a torque actuator is introduced at the top boundary of the riser to reduce the angle and transverse vibration of the riser with guaranteed closed-loop stability. In Ge, He, How, and Choo (2010), boundary control for a coupled nonlinear flexible marine riser with two actuators in transverse and longitudinal directions has been designed to suppress the riser's vibration. However, in these works, only the riser dynamics is considered and the coupling between riser and vessel is neglected, which can influence the dynamic response of the riser system and lead to an imprecise model.

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Mathematically, the flexible marine riser with vessel dynamics is represented by a set of infinite dimensional equations (i.e., PDEs describing the dynamics of the flexible riser) coupled with a set of finite dimensional equations (i.e., ODEs describing the vessel dynamics). The dynamics of the flexible mechanical system modeled by a set of PDEs is difficult to control due to the infinite dimensionality of the system. The modal control method for the control design of PDEs is based on truncated finite dimensional modes of the system, which are derived from the finite element method, the Galerkin's method or the assumed modes method (Armaou & Christofides, 2000; Balas, 1978b; Christofides & Armaou, 2000; Ge, Lee, & Zhu, 1997; Sakawa, Matsuno, & Fukushima, 1985; Vandegrift, Lewis, & Zhu, 1994). The truncated models are obtained via the model analysis or spatial discretization, in which the flexibility is represented by a finite number of modes by neglecting the higher frequency modes. The problems from the truncation procedure in the modeling need to be carefully treated in practical applications. A potential drawback in the above control design approaches is that the control can cause the actual system to become unstable due to excitation of the unmodeled, high-frequency vibration modes (i.e., spillover effects) Ge, Lee, and Zhu (1998). Spillover effects which result in instability of the system have been investigated in Balas (1978a); Meirovitch and Baruh (1983) when the control of the truncated system is restricted to a few critical modes. The control order needs to be increased with the number of flexible modes considered to achieve high accuracy of performance and the control may also be difficult to implement from the engineering point of view since full state measurements or observers are often required. In an attempt to overcome the above shortcomings of the truncated model based control, boundary control has been developed for different infinite dimensional systems (Endo, Matsuno, & Kawasaki, 2009; Ge, Lee, & Wang, 2001; Ge, Lee, & Zhu, 1996; Ge, Lee, Zhu, & Hong, 2001; Geniele, Patel, & Khorasani, 1997; How, Ge, & Choo, 2010; Karafyllis, Christofides, & Daoutidis, 1999; Krstic & Smyshlyaev, 2008; Lee, Ge, & Wang, 2001; Li, Hou, & Li, 2008; Morgul, 1992; Nguyen & Hong, 2010; Smyshlyaev, Guo, & Krstic, 2009; Yang, Hong, & Matsuno, 2004, 2005a,b; Zhu & Ge, 1998). In these papers, system dynamics analysis and control design are carried out directly based on the PDEs of the system. In contrast, boundary control where the actuation and sensing are applied only through the boundary of the system utilizes the distributed parameter model with PDEs to avoid control spillover instabilities.

Boundary control is considered to be more practical in a number of research fields including vibration control of flexible structures, fluid dynamics and heat transfer, which requires relatively few sensors and actuators. The relevant applications for this approach in mechanical flexible structures consist of second order structures (strings and cables) and fourth order structures (beams and plates) (Rahn, 2001). In Qu (2001), robust and adaptive boundary control laws are developed to reduce the vibration of a stretched string on a moving transporter. In Yang et al. (2004), adaptive boundary control is designed for an axially moving string with a spatiotemporally varying tension, where the system is proved to be asymptotically stable. In Fung and Tseng (1999), a boundary control law based on the Lyapunov method with sliding mode is employed to guarantee the asymptotic and exponential stability of an axially moving string. In Rahn, Zhang, Joshi, and Dawson (1999), boundary control for a linear gantry crane model with a flexible cable is developed and experimentally implemented. In Krstic and Smyshlyaev (2008), a backstepping boundary controller and observer are designed to stabilize the string and beam model respectively. In Baz (1997), boundary control is presented to stabilize beams by using active constrained layer damping. In Fard and Sagatun (2001), nonlinear boundary control is constructed to exponentially stabilize a free transversely vibrating beam.

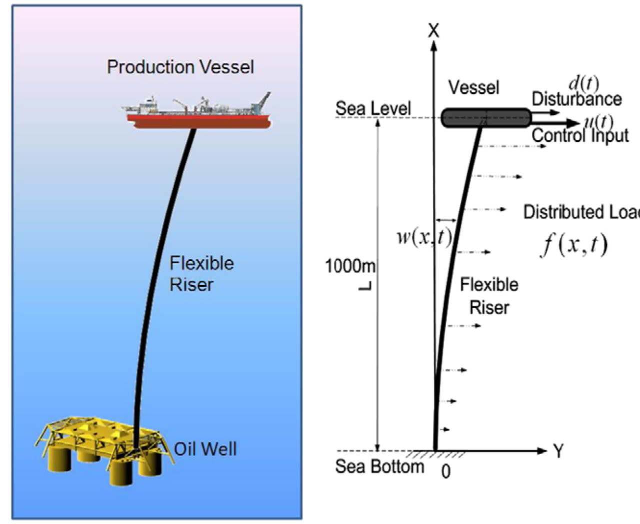

Fig. 1. A typical flexible marine riser system.

In this paper, we design the boundary control law based on the distributed parameter model of the flexible riser system. Both the dynamics of the vessel and the vibration of the riser are considered in the dynamic analysis. The stability analysis of the closed-loop system is based on Lyapunov's direct method without resorting to semigroup theory or functional analysis.

The remainder of the paper is organized as follows. The governing equation (PDE) and boundary conditions (ODEs) of the flexible riser system are introduced by use of Hamilton's principle in Section 2. The boundary control design via Lyapunov's direct method is discussed separately for both the exact model case and the system parametric uncertainty case in Section 3, where it is shown that the uniform boundedness of the closed-loop system can be achieved by the proposed control. Simulations are carried out to illustrate performance of the proposed control in Section 4. The conclusion of this paper is shown in Section 5.

2. Problem formulation and preliminaries

A typical marine riser system for crude oil transportation depicted in Fig. 1 is the connection between a production vessel on the ocean surface and a well head on the ocean floor. As shown in Fig. 1, the control is implemented from the actuator in the vessel, i.e., the top boundary of the riser. In this paper, we assume that the original position of the vessel is directly above the subsea well head with no horizontal offset and the riser is filled with seawater.

Remark 1. For clarity, the notations, $w'(x,t) = \frac{\partial w(x,t)}{\partial x}$, $w''(x,t) = \frac{\partial^2 w(x,t)}{\partial x^2}$, $w'''(x,t) = \frac{\partial^3 w(x,t)}{\partial x^3}$, $w''''(x,t) = \frac{\partial^4 w(x,t)}{\partial x^4}$, $\dot{w}(x,t) = \frac{\partial w(x,t)}{\partial t}$, and $\ddot{w}(x,t) = \frac{\partial^2 w(x,t)}{\partial t^2}$ are introduced throughout the paper.

2.1. Dynamic analysis

The kinetic energy of the riser system E_k can be represented as

$$E_k = \frac{1}{2}M_s[\dot{w}(L,t)]^2 + \frac{1}{2}\rho\int_0^L [\dot{w}(x,t)]^2 \mathrm{d}x, \tag{1}$$

where x and t represent the independent spatial and time variables respectively, M_s denotes the mass of the surface vessel, $w(L,t)$ and $\dot{w}(L,t)$ are the position and velocity of the vessel respectively, $w(x,t)$ is the displacement of the riser at the position x for time t,

$\rho > 0$ is the uniform mass per unit length of the riser, and L is the length of the riser.

The potential energy E_p of the riser system can be obtained from

$$E_p = \frac{1}{2}EI\int_0^L [w''(x,t)]^2 dx + \frac{1}{2}T\int_0^L [w'(x,t)]^2 dx, \tag{2}$$

where EI is the bending stiffness of the riser and T is the tension of the riser. The first term of Eq. (2) is due to the bending, the second term is due to the strain energy of the riser.

The virtual work done by the ocean current disturbance on the riser and the vessel is given by

$$\delta W_f = \int_0^L f(x,t)\delta w(x,t)dx + d(t)\delta w(L,t), \tag{3}$$

where $f(x,t)$ is the distributed transverse load on the riser due to the hydrodynamic effects of the ocean current, and $d(t)$ denotes the environmental disturbances on the vessel. The virtual work done by damping on the riser and the vessel is represented by

$$\delta W_d = -\int_0^L c\dot{w}(x,t)\delta w(x,t)dx - d_s\dot{w}(L,t)\delta w(L,t), \tag{4}$$

where c is the damping coefficient of the riser, and d_s denotes the damping coefficient of the vessel. We introduce the boundary control u from the actuator in the vessel, i.e., the top boundary of the riser, to produce a transverse force for vibration suppression. The virtual work done by the boundary control is written as

$$\delta W_m = u(t)\delta w(L,t). \tag{5}$$

Then, we have the total virtual work done on the system as

$$\begin{aligned}\delta W &= \delta W_f + \delta W_d + \delta W_m \\ &= \int_0^L [f(x,t) - c\dot{w}(x,t)]\,\delta w(x,t)dx \\ &\quad + [u(t) + d(t) - d_s\dot{w}(L,t)]\,\delta w(L,t).\end{aligned} \tag{6}$$

Based on the property of the Euler–Bernoulli beam for small displacement, Hamilton's principle permits the derivation of equations of motion from energy quantities in a variational form. Hamilton's principle (Goldstein, 1951) is represented by

$$\int_{t_1}^{t_2} \delta(E_k - E_p + W)dt = 0, \tag{7}$$

where t_1 and t_2 are two time instants, $t_1 < t < t_2$ is the operating interval and δ denotes the variational operator, E_k and E_p are the kinetic and potential energies of the system respectively, W denotes the virtual work done by nonconservative force acting on the system, including control force, damping and ocean disturbance. The principle states that the variation of the kinetic and potential energy plus the variation of work done by loads during any time interval $[t_1, t_2]$ must be equal to zero. Applying the variation operator and integrating Eqs. (1), (2) and (6) by parts respectively and substituting $\delta w(x,t) = 0$ at $t = t_1, t_2$, we obtain

$$\begin{aligned}\int_{t_1}^{t_2} \delta E_k dt &= -M_s\int_{t_1}^{t_2} \ddot{w}(L,t)\delta w(L,t)dt \\ &\quad - \rho\int_{t_1}^{t_2}\int_0^L \ddot{w}\delta w dx dt.\end{aligned} \tag{8}$$

Following the same procedure as in the previous equation, we have

$$\begin{aligned}&\int_{t_1}^{t_2} \delta E_p dt \\ &= \int_{t_1}^{t_2}\Bigg[EIw''\delta w'\,|_0^L - EIw'''\delta w\,|_0^L \\ &\quad + \int_0^L EIw''''\delta w dx + Tw'\delta w\Big|_0^L - \int_0^L Tw''\delta w dx\Bigg]dt,\end{aligned} \tag{9}$$

$$\begin{aligned}\int_{t_1}^{t_2} \delta W dt &= \int_{t_1}^{t_2}\int_0^L (f - c\dot{w})\,\delta w dx dt \\ &\quad + \int_{t_1}^{t_2} [u(t) + d(t) - d_s\dot{w}(L,t)]\,\delta w(L,t)dt.\end{aligned} \tag{10}$$

Substituting Eqs. (8)–(10) into the Hamilton's principle Eq. (7), we obtain the governing equations of the system as

$$\rho\ddot{w}(x,t) + EIw''''(x,t) - Tw''(x,t) - f(x,t) + c\dot{w}(x,t) = 0, \tag{11}$$

$\forall(x,t) \in (0,L) \times [0,\infty)$, and the boundary conditions of the system as

$$w'(0,t) = 0, \tag{12}$$

$$w''(L,t) = 0, \tag{13}$$

$$w(0,t) = 0, \tag{14}$$

$$\begin{aligned}&-EIw'''(L,t) + Tw'(L,t) \\ &\quad = u(t) + d(t) - d_s\dot{w}(L,t) - M_s\ddot{w}(L,t), \quad \forall t \in [0,\infty).\end{aligned} \tag{15}$$

2.2. Ocean current disturbance

The effects of a time-varying ocean current $U(x,t)$ on a riser is modeled as a distributed load (Blevins, 1977; Faltinsen, 1990). The distributed load on the flexible riser $f(x,t)$ can be expressed as a combination of a mean drag and an oscillating drag modeled as

$$f(x,t) = \frac{1}{2}\rho_s C_D(x,t)U(x,t)^2 D + A_D\cos(4\pi f_v t + \theta), \tag{16}$$

where ρ_s is the sea water density, $C_D(x,t)$ is the drag coefficient, D is the pipe outer diameter, f_v is the shedding frequency, θ is the phase angle, and A_D is the amplitude of the oscillatory part of the drag force, typically 20% of the first term in $f(x,t)$ (Faltinsen, 1990). The non-dimensional vortex shedding frequency can be expressed as

$$f_v = \frac{S_t U(x,t)}{D}, \tag{17}$$

where S_t is the Strouhal number.

Assumption 1. For the distributed load $f(x,t)$ on the riser and the environmental disturbance $d(t)$ on the vessel, we assume that there exist constants $\bar{f} \in R^+$ and $\bar{d} \in R^+$, such that $|f(x,t)| \le \bar{f}$, $\forall(x,t) \in [0,L] \times [0,\infty)$ and $|d(t)| \le \bar{d}$, $\forall(t) \in [0,\infty)$. This is a reasonable assumption as the time-varying disturbances $f(x,t)$ and $d(t)$ have finite energy and hence are bounded, i.e., $f(x,t) \in \mathcal{L}_\infty([0,L])$ and $d(t) \in \mathcal{L}_\infty$.

Remark 2. For control design in Section 3, only the assertion that there exists an upper bound on the disturbance in Assumption 1, $|f(x,t)| < \bar{f}$ and $|d(t)| \le \bar{d}$, is necessary. The knowledge of the exact values for $f(x,t)$ and $d(t)$ is not required. As such, different distributed load models up to various levels of fidelity, such as those found in Blevins (1977), Chakrabarti and Frampton (1982), Meneghini et al. (2004), Wanderley and Levi (2005) and Yamamoto, Meneghini, Saltara, Fregonesi, and Ferrari (2004) can be applied without affecting the control design or analysis.

2.3. Preliminaries

For the convenience of stability analysis, we present the following lemmas and properties for the subsequent development.

Lemma 1 (Rahn, 2001). *Let $\phi_1(x,t), \phi_2(x,t) \in R$, the following inequalities hold:*

$$\phi_1\phi_2 \le |\phi_1\phi_2| \le \phi_1^2 + \phi_2^2, \quad \forall \phi_1, \phi_2 \in R. \tag{18}$$

Lemma 2 (Rahn, 2001). *Let $\phi_1(x,t), \phi_2(x,t) \in R$, the following inequalities hold:*

$$|\phi_1\phi_2| = \left|\left(\frac{1}{\sqrt{\delta}}\phi_1\right)(\sqrt{\delta}\phi_2)\right| \le \frac{1}{\delta}\phi_1^2 + \delta\phi_2^2, \quad \forall \phi_1, \phi_2 \in R \text{ and } \delta > 0. \tag{19}$$

Lemma 3 (Hardy, Littlewood, & Polya, 1959). *Let $\phi(x,t) \in R$ be a function defined on $x \in [0,L]$ and $t \in [0,\infty)$ that satisfies the boundary condition*

$$\phi(0,t) = 0, \quad \forall t \in [0,\infty), \tag{20}$$

then the following inequalities hold:

$$\phi^2 \le L\int_0^L [\phi']^2 \mathrm{d}x. \tag{21}$$

Property 1 (Queiroz, Dawson, Nagarkatti, & Zhang, 2000). *If the kinetic energy of the system (11)–(15), given by Eq. (1) is bounded $\forall t \in [0,\infty)$, then $\dot{w}(x,t)$, $\dot{w}'(x,t)$, $\dot{w}''(x,t)$ and $\dot{w}'''(x,t)$ are bounded $\forall (x,t) \in [0,L]\times[0,\infty)$.*

Property 2 (Queiroz et al., 2000). *If the potential energy of the system (11)–(15), given by Eq. (2) is bounded $\forall t \in [0,\infty)$, then $w''(x,t)$, $w'''(x,t)$ and $w''''(x,t)$ are bounded $\forall (x,t) \in [0,L] \times [0,\infty)$.*

3. Control design

The control objective is to suppress the vibration of the riser and stabilize the riser at the small neighborhood of its original position in the presence of the time-varying distributed load $f(x,t)$ and the disturbance $d(t)$ due to the ocean current. In this section, Lyapunov's direct method is used to construct a boundary control law $u(t)$ at the top boundary of the riser and to analyze the closed-loop stability of the system.

In this paper, we analyze two cases for the riser system: (i) exact model-based control, i.e., EI, T, M_s and d_s are all known; and (ii) adaptive control for the system parametric uncertainty, i.e., EI, T, M_s and d_s are unknown. For the first case, robust boundary control is introduced for the exact model of the riser system subject to the ocean disturbance. For second case where the system parameters cannot be directly measured, the adaptive control is designed to compensate the system parametric uncertainty.

3.1. Robust boundary control based on exact model of the riser system

To stabilize the system given by governing Eq. (11) and boundary Eqs. (12)–(15), we propose the following control law:

$$\begin{aligned} u = {} & -EIw'''(L,t) + Tw'(L,t) - \mathrm{sgn}(u_a)\bar{d} + d_s\dot{w}(L,t) \\ & - k_1M_s\dot{w}'(L,t) + k_2M_s\dot{w}'''(L,t) - ku_a, \end{aligned} \tag{22}$$

where $\mathrm{sgn}(\cdot)$ denotes the signum function, k, k_1, k_2 are the control gains and the auxiliary signal u_a is defined as

$$u_a = \dot{w}(L,t) + k_1w'(L,t) - k_2w'''(L,t). \tag{23}$$

After differentiating the auxiliary signal Eq. (23), multiplying the resulting equation by M_s, and substituting Eq. (15), we obtain

$$\begin{aligned} M_s\dot{u}_a = {} & EIw'''(L,t) - Tw'(L,t) + d - d_s\dot{w}(L,t) \\ & + k_1M_s\dot{w}'(L,t) - k_2M_s\dot{w}'''(L,t) + u. \end{aligned} \tag{24}$$

Substituting Eq. (22) into Eq. (24), we have

$$M_s\dot{u}_a = -ku_a + d - \mathrm{sgn}(u_a)\bar{d}. \tag{25}$$

Remark 3. All the signals in the boundary control can be measured by sensors or obtained by a backward difference algorithm. $w(L,t)$ can be sensed by a laser displacement sensor at the top boundary of the riser, $w'(L,t)$ can be measured by an inclinometer and $w'''(L,t)$ can be obtained by a shear force sensor. In practice, the effect of measurement noise from sensors is unavoidable, which will affect the control implementation, especially when the high order differentiating terms with respect to time exist. In our proposed control (22), $\dot{w}(L,t)$, $\dot{w}'(L,t)$ and $\dot{w}'''(L,t)$ with only differentiating once with respect to time can be calculated with a backward difference algorithm. It is noted that differentiating twice and thrice the position $w(L,t)$ with respect to time to get $\ddot{w}(L,t)$ and $\dddot{w}(L,t)$ respectively, are undesirable in practice due to noise amplification. For these cases, observers are needed to design to estimate the states values according to the boundary conditions.

Remark 4. The control design is based on the distributed parameter model Eqs. (11)–(15), and the spillover problems associated with traditional truncated model-based approaches caused by ignoring high-frequency modes in controller and observer design are avoided. For results on model-based control of a distributed parameter system which is helpful in avoiding spillover effects, the readers can refer to Armaou and Christofides (2000) and Christofides and Armaou (2000).

Consider the Lyapunov function candidate

$$V = V_1 + V_2 + V_3, \tag{26}$$

where the energy term V_1 and an auxiliary term V_2 and a small crossing term V_3 are defined as

$$\begin{aligned} V_1 = {} & \frac{\beta k_2}{2}\rho\int_0^L [\dot{w}]^2\mathrm{d}x + \frac{\beta k_2}{2}EI\int_0^L [w'']^2\mathrm{d}x \\ & + \frac{\beta k_2}{2}T\int_0^L [w']^2\mathrm{d}x, \end{aligned} \tag{27}$$

$$V_2 = \frac{1}{2}M_su_a^2, \tag{28}$$

$$V_3 = \alpha\rho\int_0^L x\dot{w}w'\mathrm{d}x, \tag{29}$$

where k_2 is the control gain, and α, β are the two positive weighting constants.

Lemma 4. *The Lyapunov function candidate given by (26) is upper and lower bounded as*

$$0 \le \lambda_1(V_1 + V_2) \le V \le \lambda_2(V_1 + V_2), \tag{30}$$

where λ_1 and λ_2 are two positive constants defined as

$$\lambda_1 = 1 - \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta Tk_2)}, \tag{31}$$

$$\lambda_2 = 1 + \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta Tk_2)}. \tag{32}$$

Proof. See Appendix A. □

Lemma 5. *The time derivative of the Lyapunov function candidate (26) is upper bounded with*

$$\dot{V} \leq -\lambda V + \varepsilon, \tag{33}$$

where λ and ε are defined in Appendix B.

Proof. See Appendix B. □

With the above lemmas, the exact model-based control design for riser system subjected to the ocean current disturbance can be summarized in the following theorem.

Theorem 1. *For the system dynamics described by (11) and boundary conditions (12)–(15), under Assumption 1, and the control law (22), given that the initial conditions are bounded, we can conclude that uniform boundedness (UB): the state of the closed loop system $w(x,t)$ will remain in the compact set Ω defined by*

$$\Omega := \{w(x,t) \in R \,|\, |w(x,t)| \leq D_1, \forall (x,t) \in [0,L] \times [0,\infty)\}, \tag{34}$$

where constant $D_1 = \sqrt{\frac{2L}{\beta T \lambda_1 k_2}\left(V(0) + \frac{\varepsilon}{\lambda}\right)}$.

Proof. Multiplying Eq. (33) by $e^{\lambda t}$ yields

$$\frac{\partial}{\partial t}(V e^{\lambda t}) \leq \varepsilon e^{\lambda t}. \tag{35}$$

Integration of the above inequality, we obtain

$$V \leq \left(V(0) - \frac{\varepsilon}{\lambda}\right) e^{-\lambda t} + \frac{\varepsilon}{\lambda} \leq V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda} \in \mathcal{L}_\infty, \tag{36}$$

which implies V is bounded. Utilizing Ineq. (21) and Eq. (27), we have

$$\begin{aligned} \frac{\beta k_2}{2L} T w^2(x,t) &\leq \frac{\beta k_2}{2} T \int_0^L [w'(x,t)]^2 dx \leq V_1 \\ &\leq V_1 + V_2 \leq \frac{1}{\lambda_1} V \in \mathcal{L}_\infty. \end{aligned} \tag{37}$$

Appropriately rearranging the terms of the above inequality, we obtain $w(x,t)$ is uniformly bounded as follows:

$$\begin{aligned} |w(x,t)| &\leq \sqrt{\frac{2L}{\beta T \lambda_1 k_2}\left(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda}\right)} \\ &\leq \sqrt{\frac{2L}{\beta T \lambda_1 k_2}\left(V(0) + \frac{\varepsilon}{\lambda}\right)}, \\ &\quad \forall (x,t) \in [0,L] \times [0,\infty). \quad \square \end{aligned} \tag{38}$$

Remark 5. By choosing the proper values of α and β in Appendix B, it is shown that the increase in the control gain k will result in a larger σ_4, which will lead to a greater λ_3. Then the value of λ will increase, which will reduce the size of Ω and produce a better vibration suppression performance. We can conclude that the bound of the system state $w(x,t)$ can be made arbitrarily small provided that the design control parameters are appropriately selected. However, increasing k will bring a high gain control problem. Therefore, in practical applications, the design parameters should be adjusted carefully for achieving suitable transient performance and control action.

Remark 6. From Eq. (37), we can state that V_1 is bounded $\forall t \in [0,\infty)$. Since V_1 is bounded, $\dot{w}(x,t)$, $w''(x,t)$ and $w'(x,t)$ are bounded $\forall (x,t) \in [0,L] \times [0,\infty)$. From Eq. (1), the kinetic energy of the system is bounded and using Property 1, $\dot{w}'(x,t)$ and $\dot{w}'''(x,t)$ are also bounded $\forall (x,t) \in [0,L] \times [0,\infty)$. From the boundedness of the potential energy Eq. (2), we can use Property 2 to obtain that $w'''(x,t)$ and $w''''(x,t)$ are bounded. Using Assumption 1, Eq. (11) and the above statements, we can state that $\ddot{w}(x,t)$ is also bounded $\forall (x,t) \in [0,L] \times [0,\infty)$. From the above information, it is shown that the proposed control Eq. (22) ensures all internal system signals including $w(x,t)$, $w'(x,t)$, $\dot{w}(x,t)$, $\dot{w}'(x,t)$, $\ddot{w}(x,t)$, $w'''(x,t)$, $\dot{w}'''(x,t)$ and $w''''(x,t)$ are uniformly bounded. Since $\dot{w}(x,t)$, $w'(x,t)$, $\dot{w}'(x,t)$, $w'''(x,t)$ and $\dot{w}'''(x,t)$ are all bounded $\forall (x,t) \in [0,L] \times [0,\infty)$, and we can conclude the boundary control Eq. (22) is also bounded $\forall t \in [0,\infty)$.

Remark 7. For the system dynamics described by Eq. (11) and boundary conditions (12)–(15), if $f(x,t) = 0$, the exponential stability can be achieved with the proposed boundary control (22) as follows:

$$|w(x,t)| \leq \sqrt{\frac{2L}{\beta T \lambda_1 k_2} V(0) e^{-\lambda t}}, \quad \forall (x,t) \in [0,L] \times [0,\infty). \tag{39}$$

3.2. Adaptive boundary control for parametric uncertainty

In Section 3.1, the exact model-based boundary control Eq. (22) requires the exact knowledge of the riser system. Adaptive boundary control is designed to improve the performance of the system via parameter estimation when there are some unknown parameters. The exact model-based boundary control provides a stepping stone towards adaptive control, which is designed to deal with the system parametric uncertainty. In this section, the boundary control Eq. (22) is redesigned by using adaptive control since EI, T, d_s and M_s are unknown. We rewrite Eq. (24) in the following form

$$M_s \dot{u}_a = P\Phi + d + u, \tag{40}$$

where vectors P and Φ are defined as

$$P = [w'''(L,t) \quad -w'(L,t) \quad -\dot{w}(L,t) \quad k_1\dot{w}'(L,t) - k_2\dot{w}'''(L,t)], \tag{41}$$

$$\Phi = [EI \quad T \quad d_s \quad M_s]^T. \tag{42}$$

We propose the following adaptive boundary control law for system

$$u = -P\hat{\Phi} - ku_a - \mathrm{sgn}(u_a)\bar{d}, \tag{43}$$

where the parameter estimate vector $\hat{\Phi}$ is defined as

$$\hat{\Phi} = [\widehat{EI} \quad \widehat{T} \quad \widehat{d_s} \quad \widehat{M_s}]^T. \tag{44}$$

The adaptation law is designed as

$$\dot{\hat{\Phi}} = \Gamma P^T u_a - r\Gamma\hat{\Phi}, \tag{45}$$

where $\Gamma \in R^{4\times 4}$ is a diagonal positive-definite matrix and r is a positive constant. We define the maximum and minimum eigenvalue of matrix Γ as $\lambda_{\max}$ and $\lambda_{\min}$ respectively. The parameter estimate error vector $\tilde{\Phi} \in R^4$ is defined as

$$\tilde{\Phi} = \Phi - \hat{\Phi}. \tag{46}$$

Substituting Eq. (43) into Eq. (40) and using Eq. (46) in Eq. (45), we have

$$M_s \dot{u}_a = P\tilde{\Phi} - ku_a + d - \mathrm{sgn}(u_a)\bar{d}, \tag{47}$$

$$\dot{\tilde{\Phi}} = -\Gamma P^T u_a + r\Gamma\hat{\Phi}. \tag{48}$$

Consider the Lyapunov function candidate

$$V_a = V + \frac{1}{2}\tilde{\Phi}^T \Gamma^{-1} \tilde{\Phi}, \tag{49}$$

where V is defined as Eq. (26), and $\tilde{\Phi}$ is the parameter estimate error vector.

Lemma 6. *The Lyapunov function candidate given by (49) is upper and lower bounded as*

$$0 \le \lambda_{1a}(V_1 + V_2 + \|\tilde{\Phi}\|^2) \le V_a \le \lambda_{2a}(V_1 + V_2 + \|\tilde{\Phi}\|^2), \tag{50}$$

where λ_{1a} and λ_{2a} are two positive constants defined as

$$\lambda_{1a} = \min\left(1 - \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)}, \frac{1}{2\lambda_{\max}}\right), \tag{51}$$

$$\lambda_{2a} = \max\left(1 + \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)}, \frac{1}{2\lambda_{\min}}\right). \tag{52}$$

Proof. See Appendix C. □

Lemma 7. *The time derivative of the Lyapunov function candidate (49) is upper bounded with*

$$\dot{V}_a \le -\lambda_a V_a + \psi, \tag{53}$$

where λ_a and ψ are two positive constants defined in Appendix D.

Proof. See Appendix D. □

With the above lemmas, the adaptive control design for the riser system subjected to the ocean current disturbance can be summarized in the following theorem.

Theorem 2. *For the system dynamics described by (11) and boundary conditions (12)–(15), under Assumption 1, and the control law (43), given that the initial conditions are bounded, we can conclude that uniform boundedness (UB): the state of the closed loop system $w(x,t)$ will remain in the compact set Ω_a defined by*

$$\Omega_a := \{w(x,t) \in R \mid |w(x,t)| \le D_2, \forall (x,t) \in [0,L] \times [0,\infty)\}, \tag{54}$$

where constant $D_2 = \sqrt{\frac{2L}{\beta T \lambda_{1a} k_2}\left(V_a(0) + \frac{\psi}{\lambda_a}\right)}$.

Proof. Multiplying Eq. (53) by $e^{\lambda_a t}$ yields

$$\frac{\partial}{\partial t}(V_a e^{\lambda_a t}) \le \psi e^{\lambda_a t}. \tag{55}$$

Integrating of the above inequality, we obtain

$$V_a \le \left(V_a(0) - \frac{\psi}{\lambda_a}\right)e^{-\lambda_a t} + \frac{\psi}{\lambda_a} \le V_a(0)e^{-\lambda_a t} + \frac{\psi}{\lambda_a}, \tag{56}$$

which implies V_a is bounded. Utilizing Ineq. (21) and Eq. (27), we have

$$\begin{aligned}\frac{\beta k_2}{2L} T w^2(x,t) &\le \frac{\beta k_2}{2} T \int_0^L [w'(x,t)]^2 dx \le V_1 \le V_1 + V_2 \\ &\le \frac{1}{\lambda_{1a}} V_a \in \mathcal{L}_\infty.\end{aligned} \tag{57}$$

Appropriately rearranging the terms of the above inequality, we obtain $w(x,t)$ is uniformly bounded as follows:

$$\begin{aligned}|w(x,t)| &\le \sqrt{\frac{2L}{\beta T \lambda_{1a} k_2}\left(V_a(0)e^{-\lambda_a t} + \frac{\psi}{\lambda_a}\right)} \\ &\le \sqrt{\frac{2L}{\beta T \lambda_{1a} k_2}\left(V_a(0) + \frac{\psi}{\lambda_a}\right)}, \\ &\forall (x,t) \in [0,L] \times [0,\infty). \quad \Box\end{aligned} \tag{58}$$

Table 1
Parameters of the riser system.

Parameter	Description	Value
L	Riser length	1000.00 m
D	Riser external diameter	152.40 mm
EI	Riser stiffness	1.5×10^7 N m^2
M_s	Vessel mass	9.60×10^6 kg
d_s	Vessel damping	1×10^3 N s/m
T	Riser tension	8.11×10^7 N
ρ	Riser mass per unit	500.00 kg/m
ρ_s	Sea water density	1024.00 kg/m^3
c	Riser damping	2.00 N s/m^2

Remark 8. From a similar analysis of Remark 5, we can conclude that system state $w(x,t)$ with the proposed robust adaptive boundary control can be made arbitrarily small by choosing control gain k in Eq. (43) appropriately.

Remark 9. From Eq. (56), we can obtain that the parameter estimate error $\tilde{\Phi}$ is bounded $\forall t \in [0,\infty)$. Using a derivation similar to those employed in Remark 6, we can state the proposed control Eq. (43) ensures all internal system signals including $w(x,t)$, $w'(x,t)$, $\dot{w}(x,t)$, $\dot{w}'(x,t)$, $\ddot{w}(x,t)$, $w'''(x,t)$, $\dot{w}'''(x,t)$ and $w''''(x,t)$ are uniformly bounded. Since $\hat{\Phi}$, $w'(x,t)$, $\dot{w}(x,t)$, $w'''(x,t)$ and $\dot{w}'''(x,t)$ are all bounded $\forall (x,t) \in [0,L] \times [0,\infty)$, and we can conclude the boundary adaptive control Eq. (43) is also bounded $\forall t \in [0,\infty)$.

Remark 10. For the system dynamics described by Eq. (11) and boundary conditions (12)–(15), if there is no distributed disturbance for the riser system, i.e., $f(x,t) = 0$, the boundedness stability can be achieved with the proposed boundary control (43) as follows:

$$\begin{aligned}|w(x,t)| \le &\sqrt{\frac{2L}{\beta T \lambda_{1a} k_2}\left(V_a(0)e^{-\lambda_a t} + \frac{r\|\Phi\|^2}{2\lambda_a}\right)}, \\ &\forall (x,t) \in [0,L] \times [0,\infty).\end{aligned} \tag{59}$$

4. Numerical simulations

Simulations for a riser of length 1000 m under the ocean current disturbance are carried out to demonstrate the effectiveness of the proposed boundary control Eqs. (22) and (43). In this article, the finite difference (FD) method is chosen to simulate the system performance with the proposed boundary control.

The riser, initially at rest, is excited by a distributed transverse disturbance due to the ocean current. The corresponding initial conditions of the riser system are given as

$$w(x,0) = 0, \tag{60}$$

$$\dot{w}(x,0) = 0. \tag{61}$$

The system parameters are given in Table 1.

Large vibrational stresses are normally associated with a resonance that exists when the frequency of the imposed force is tuned to one of the natural frequencies (Bokaian, 1990). In our simulation experiments, the ocean surface current velocity $U(t)$ is modeled as a mean flow with worst case sinusoidal components to simulate the riser with a mean deflected profile. The sinusoids have frequencies of $\omega_i = \{0.867, 1.827, 2.946, 4.282\}$, for $i = 1$–4, corresponding to the four natural modes of vibration of the riser. The current $U(t)$ is expressed as

$$U(t) = \bar{U} + U' \sum_{i=1}^{4} \sin(\omega_i t), \quad i = 1, 2, \ldots, 4, \tag{62}$$

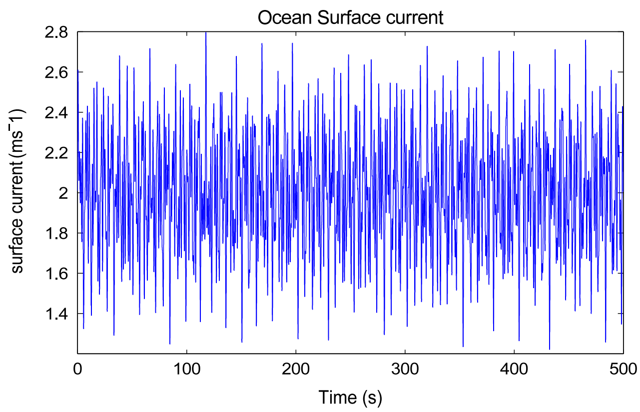

Fig. 2. Ocean surface current $U(t)$.

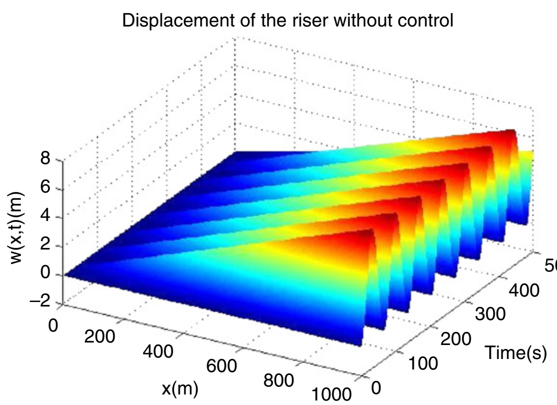

Fig. 3. Displacement of the riser without control.

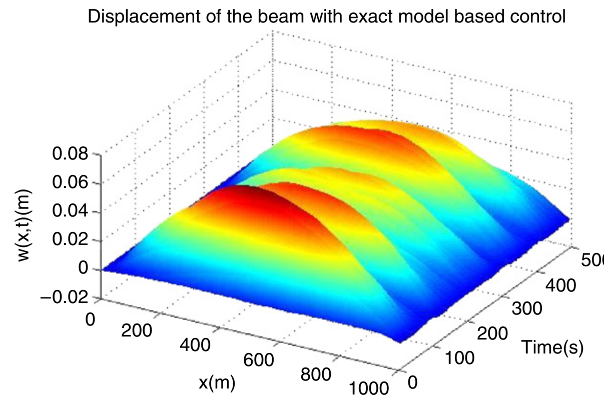

Fig. 4. Displacement of the riser with exact model-based control.

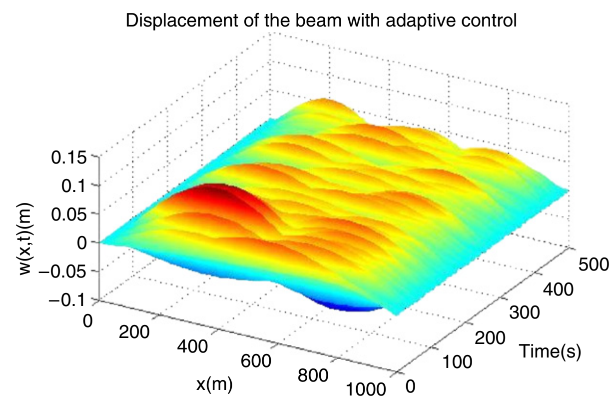

Fig. 5. Displacement of the riser with adaptive control.

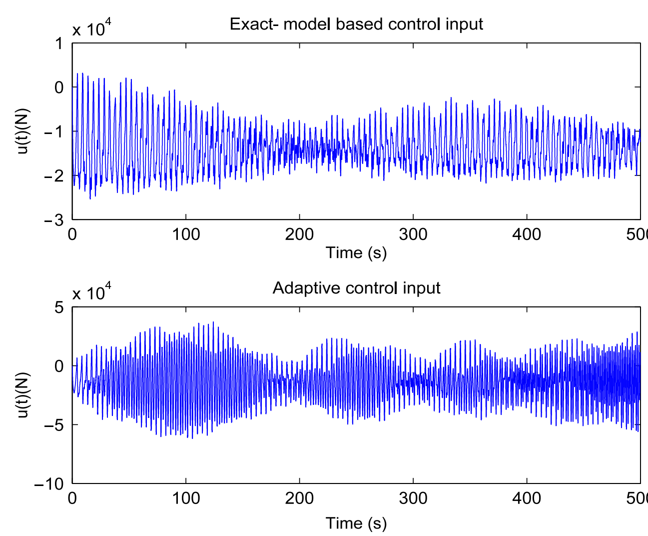

Fig. 6. Control input $u(t)$.

where $\bar{U} = 2\ \mathrm{ms}^{-1}$ is the mean flow current and $U' = 0.2$ is the amplitude of the oscillating flow. The surface current generated by Eq. (62) is shown in Fig. 2. The full current load is applied from $x = 1000$ m to $x = 0$ m and thereafter linearly decline to zero at the ocean floor, $x = 0$, to obtain a depth dependent ocean current profile $U(x, t)$. The distributed load $f(x, t)$ is generated using Eq. (16) with $C_D = 1.361$, $\beta = 0$, $S_t = 0.2$ and $f_v = 2.625$. The disturbance $d(t)$ on the vessel is generated by the following equation.

$$d(t) = [3 + 0.8\sin(0.7t) + 0.2\sin(0.5t) + 0.2\sin(0.9t)] \times 10^5. \tag{63}$$

Displacement of the riser system for free vibration, i.e., $u(t) = 0$, under the ocean disturbance is shown in Fig. 3. Displacement of the riser system with exact model-based control Eq. (22), by choosing $k = 1 \times 10^7$, under the ocean disturbance is shown in Fig. 4. When the system parameters EI, T, d_s and M_s are unknown, displacement of the riser system with adaptive control Eq. (43), by choosing $k = 1 \times 10^7$, $r = 0.0001$ and $\Gamma = \mathrm{diag}\{1, 1, 1, 1\}$, under the ocean disturbance is shown in Fig. 5. Figs. 4 and 5 illustrate that the proposed boundary control (22) and (43) are able to stabilize the riser at the small neighborhood of zero by appropriately choosing design parameters. The corresponding boundary control input for the exact model-based control and the adaptive control are shown in Fig. 6.

5. Conclusion

Vibration suppression for a flexible marine riser system subjected to ocean current disturbance has been presented in this paper. Two cases have been investigated: (i) exact model-based control, and (ii) robust adaptive control for the system parametric uncertainty. Robust boundary control has been proposed based on the exact model of the riser system, and adaptive control has been designed to compensate the system parametric uncertainty. With the proposed control, closed-loop stability under the external disturbance has been proven by using Lyapunov's direct method. The proposed control is designed based on the original infinite dimensional model (PDE), and the spillover instability phenomenon is eliminated. The control is implementable since all the required signals in the control can be measured by sensors or obtained by a backward difference algorithm. Numerical simulations have been provided to illustrate the effectiveness of the proposed boundary control.

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Appendix A. Proof of Lemma 4

Proof. Applying Ineq. (18) in Eq. (29) yields

$$|V_3| \le \alpha\rho L \int_0^L ([w']^2 + [\dot{w}]^2)\mathrm{d}x \le \alpha_1 V_1, \tag{64}$$

where

$$\alpha_1 = \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)}. \tag{65}$$

Then, we obtain

$$-\alpha_1 V_1 \le V_3 \le \alpha_1 V_1. \tag{66}$$

Considering α is a small positive weighting constant satisfying $0 < \alpha < \frac{\min(\beta\rho k_2, \beta T k_2)}{2\rho L}$, we can obtain

$$\alpha_2 = 1 - \alpha_1 = 1 - \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)} > 0, \tag{67}$$

$$\alpha_3 = 1 + \alpha_1 = 1 + \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)} > 1. \tag{68}$$

Then, we further have

$$0 \le \alpha_2 V_1 \le V_1 + V_3 \le \alpha_3 V_1. \tag{69}$$

Given the Lyapunov function candidate in Eq. (26), we obtain

$$0 \le \lambda_1(V_1 + V_2) \le V \le \lambda_2(V_1 + V_2), \tag{70}$$

where $\lambda_1 = \min(\alpha_2, 1) = \alpha_2$ and $\lambda_2 = \max(\alpha_3, 1) = \alpha_3$ are two positive constants. □

Appendix B. Proof of Lemma 5

Proof. Differentiating Eq. (26) with respect to time leads to

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3. \tag{71}$$

The first term of the Eq. (71) is

$$\dot{V}_1 = A_1 + A_2 + A_3, \tag{72}$$

where

$$A_1 = \beta\rho k_2 \int_0^L \dot{w}\ddot{w}\mathrm{d}x, \tag{73}$$

$$A_2 = \beta EIk_2 \int_0^L w''\dot{w}''\mathrm{d}x, \tag{74}$$

$$A_3 = \beta T k_2 \int_0^L w'\dot{w}'\mathrm{d}x. \tag{75}$$

Substituting the governing equation (11) into A_1, we obtain

$$A_1 = \beta k_2 \int_0^L \dot{w}\left(-EIw'''' + Tw'' + f - c\dot{w}\right)\mathrm{d}x. \tag{76}$$

Using the boundary conditions and integrating Eq. (74) by parts, we obtain

$$A_2 = -\beta EIk_2 w'''(L,t)\dot{w}(L,t) + \beta EIk_2 \int_0^L \dot{w}w''''\mathrm{d}x. \tag{77}$$

Using the boundary conditions and integrating Eq. (75) by parts, we obtain

$$A_3 = \beta T k_2 w'(L,t)\dot{w}(L,t) - \beta T k_2 \int_0^L \dot{w}w''\mathrm{d}x. \tag{78}$$

Substituting Eqs. (76)–(78) into Eq. (72), we have

$$\begin{aligned}\dot{V}_1 &= \beta k_2\left[-EIw'''(L,t) + Tw'(L,t)\right]\dot{w}(L,t) \\ &\quad - \beta c k_2 \int_0^L [\dot{w}]^2\mathrm{d}x + \beta k_2 \int_0^L f\dot{w}\mathrm{d}x.\end{aligned} \tag{79}$$

Substituting the Eq. (23) into Ineq. (79), we obtain

$$\begin{aligned}\dot{V}_1 &= -\frac{\beta EI}{2}\left[[\dot{w}(L,t)]^2 + k_2^2[w'''(L,t)]^2 + k_1^2[w'(L,t)]^2\right] \\ &\quad + \frac{\beta EI}{2}u_a^2 + \beta(Tk_2 - EIk_1)w'(L,t)\dot{w}(L,t) \\ &\quad + \beta EIk_1k_2 w'''(L,t)w'(L,t) - \beta c k_2 \int_0^L [\dot{w}]^2\mathrm{d}x \\ &\quad + \beta k_2 \int_0^L f\dot{w}\mathrm{d}x.\end{aligned} \tag{80}$$

Using Ineq. (19), we obtain

$$\begin{aligned}\dot{V}_1 &\le -\frac{\beta EI}{2}\left[[\dot{w}(L,t)]^2 + k_2^2[w'''(L,t)]^2 + k_1^2[w'(L,t)]^2\right] \\ &\quad + \frac{\beta EI}{2}u_a^2 + \beta|Tk_2 - EIk_1|\delta_1[w'(L,t)]^2 \\ &\quad + \frac{\beta}{\delta_1}|Tk_2 - EIk_1|[\dot{w}(L,t)]^2 - \beta(c - \delta_2)k_2 \int_0^L [\dot{w}]^2\mathrm{d}x \\ &\quad + \beta EIk_1k_2 w'''(L,t)w'(L,t) + \frac{\beta k_2}{\delta_2}\int_0^L f^2\mathrm{d}x,\end{aligned} \tag{81}$$

where δ_1 and δ_2 are two positive constants.

The second term of the Eq. (71) is

$$\begin{aligned}\dot{V}_2 &= M_s u_a \dot{u}_a \\ &= -ku_a^2 + \mathrm{d}u_a - \mathrm{sgn}(u_a)u_a\bar{d} \\ &= -ku_a^2 + \mathrm{d}u_a - |u_a|\bar{d} \\ &\le -ku_a^2.\end{aligned} \tag{82}$$

The third term of the Eq. (71) is

$$\begin{aligned}\dot{V}_3 &= \alpha\rho \int_0^L (x\ddot{w}w' + x\dot{w}\dot{w}')\mathrm{d}x \\ &= \alpha \int_0^L xw'\left[-EIw'''' + Tw'' + f - c\dot{w}\right]\mathrm{d}x \\ &\quad + \alpha\rho \int_0^L x\dot{w}\dot{w}'\mathrm{d}x \\ &= B_1 + B_2 + B_3 + B_4 + B_5,\end{aligned} \tag{83}$$

where

$$B_1 = -\alpha \int_0^L EIxw'w''''\mathrm{d}x, \tag{84}$$

$$B_2 = \alpha \int_0^L Txw'w''\mathrm{d}x, \tag{85}$$

$$B_3 = \alpha \int_0^L fxw'\mathrm{d}x, \tag{86}$$

$$B_4 = -\alpha \int_0^L cxw'\dot{w}\mathrm{d}x, \tag{87}$$

$$B_5 = \alpha\rho \int_0^L x\dot{w}\dot{w}'\mathrm{d}x. \tag{88}$$

After integrating Eq. (84) by parts and using the boundary conditions, we obtain

$$B_1 = -\alpha EILw'(L,t)w'''(L,t) + \alpha EI \int_0^L w'w'''\mathrm{d}x + \alpha EI \int_0^L xw''w'''\mathrm{d}x. \tag{89}$$

By integrating Eq. (89) by parts, we obtain

$$B_1 = -\alpha EILw'(L,t)w'''(L,t) - \frac{3\alpha EI}{2}\int_0^L [w'']^2\mathrm{d}x. \tag{90}$$

After integrating Eq. (85) by parts and using the boundary conditions, we obtain

$$B_2 = \alpha TL[w'(L,t)]^2 - \alpha T \int_0^L \left([w']^2 + xw'w''\right)\mathrm{d}x. \tag{91}$$

Combining Eqs. (85) and (91), we obtain

$$B_2 = \frac{\alpha TL}{2}[w'(L,t)]^2 - \frac{\alpha T}{2}\int_0^L [w']^2\mathrm{d}x. \tag{92}$$

Using Ineq. (19), we obtain

$$B_3 \le \frac{\alpha L}{\delta_3}\int_0^L f^2\mathrm{d}x + \alpha L\delta_3 \int_0^L [w']^2\mathrm{d}x, \tag{93}$$

$$B_4 \le \frac{\alpha cL}{\delta_4}\int_0^L [\dot{w}]^2\mathrm{d}x + \alpha cL\delta_4 \int_0^L [w']^2\mathrm{d}x, \tag{94}$$

where δ_3 and δ_4 are two positive constants. Integrating Eq. (88) by parts, we obtain

$$B_5 = \frac{\alpha\rho L}{2}[\dot{w}(L,t)]^2 - \frac{\alpha\rho}{2}\int_0^L [\dot{w}]^2\mathrm{d}x. \tag{95}$$

Applying Eqs. (90), (95) and Ineqs. (92)–(94) in Eq. (83), we obtain

$$\begin{aligned}\dot{V}_3 \le{}& -\alpha EILw'(L,t)w'''(L,t) - \frac{3\alpha EI}{2}\int_0^L [w'']^2\mathrm{d}x \\ &+ \frac{\alpha TL}{2}[w'(L,t)]^2 - \frac{\alpha T}{2}\int_0^L [w']^2\mathrm{d}x + \frac{\alpha L}{\delta_3}\int_0^L f^2\mathrm{d}x \\ &+ \alpha L\delta_3 \int_0^L [w']^2\mathrm{d}x + \frac{\alpha cL}{\delta_4}\int_0^L [\dot{w}]^2\mathrm{d}x + \alpha cL\delta_4 \int_0^L [w']^2\mathrm{d}x \\ &+ \frac{\alpha\rho L}{2}[\dot{w}(L,t)]^2 - \frac{\alpha\rho}{2}\int_0^L [\dot{w}]^2\mathrm{d}x. \end{aligned} \tag{96}$$

Applying Ineqs. (81), (82) and (96) into Eq. (26), and utilizing Ineqs. (19), we obtain

$$\begin{aligned}\dot{V} \le{}& -\left(\beta ck_2 + \frac{\alpha\rho}{2} - \beta\delta_2 k_2 - \frac{\alpha cL}{\delta_4}\right)\int_0^L [\dot{w}]^2\mathrm{d}x \\ &- \left(\frac{\alpha T}{2} - \alpha L\delta_3 - \alpha cL\delta_4\right)\int_0^L [w']^2\mathrm{d}x \\ &- \left(\frac{\beta EIk_1^2}{2} - |\beta EIk_1k_2 - \alpha EIL|\delta_5 - \frac{\alpha TL}{2}\right. \\ &\left. - \beta|Tk_2 - EIk_1|\delta_1\right)[w'(L,t)]^2 - \left(k - \frac{\beta EI}{2}\right)u_a^2 \\ &- \left(\frac{\beta EI}{2} - \frac{\beta}{\delta_1}|Tk_2 - EIk_1| - \frac{\alpha\rho L}{2}\right)[\dot{w}(L,t)]^2 \\ &- \left(\frac{\beta EIk_2^2}{2} - \frac{|\beta EIk_1k_2 - \alpha EIL|}{\delta_5}\right)[w'''(L,t)]^2 \\ &- \frac{3\alpha EI}{2}\int_0^L [w'']^2\mathrm{d}x + \left(\frac{\beta k_2}{\delta_2} + \frac{\alpha L}{\delta_3}\right)\int_0^L \bar{f}^2\mathrm{d}x \\ \le{}& -\lambda_3(V_1 + V_2) + \varepsilon, \end{aligned} \tag{97}$$

where $\varepsilon = \left(\frac{\beta k_2}{\delta_2} + \frac{\alpha L}{\delta_3}\right)\int_0^L \bar{f}^2\mathrm{d}x = \left(\frac{\beta k_2}{\delta_2} + \frac{\alpha L}{\delta_3}\right)L\bar{f}^2$, the constants $k, k_1, k_2, \alpha, \beta, \delta_1, \delta_2, \delta_3, \delta_4$ and δ_5 are chosen to satisfy the following conditions:

$$\alpha < \frac{\min(\beta\rho k_2, \beta Tk_2)}{2\rho L}, \tag{98}$$

$$\frac{\beta EIk_1^2}{2} - |\beta EIk_1k_2 - \alpha EIL|\delta_5 - \frac{\alpha TL}{2} - \beta|Tk_2 - EIk_1|\delta_1 \ge 0, \tag{99}$$

$$\frac{\beta EI}{2} - \frac{\beta}{\delta_1}|Tk_2 - EIk_1| - \frac{\alpha\rho L}{2} \ge 0, \tag{100}$$

$$\frac{\beta EIk_2^2}{2} - \frac{|\beta EIk_1k_2 - \alpha EIL|}{\delta_5} \ge 0, \tag{101}$$

$$\sigma_1 = \beta ck_2 + \frac{\alpha\rho}{2} - \beta\delta_2 k_2 - \frac{\alpha cL}{\delta_4} > 0, \tag{102}$$

$$\sigma_2 = \frac{3\alpha EI}{2} > 0, \tag{103}$$

$$\sigma_3 = \frac{\alpha T}{2} - \alpha L\delta_3 - \alpha cL\delta_4 > 0, \tag{104}$$

$$\sigma_4 = k - \frac{\beta EI}{2} > 0, \tag{105}$$

$$\lambda_3 = \min\left(\frac{2\sigma_1}{\beta\rho}, \frac{2\sigma_2}{\beta EI}, \frac{2\sigma_3}{\beta T}, \frac{2\sigma_4}{M_s}\right) > 0. \tag{106}$$

From Ineqs. (70) and (97) we have

$$\dot{V} \le -\lambda V + \varepsilon, \tag{107}$$

where $\lambda = \lambda_3/\lambda_2$ and ε are two positive constants. □

Appendix C. Proof of Lemma 6

Proof. From Ineq. (30), we have

$$\lambda_1(V_1 + V_2) \le V \le \lambda_2(V_1 + V_2), \tag{108}$$

where λ_1 and λ_2 are two positive constants defined in Eqs. (31) and (32). From the properties of matrix Γ, we have

$$\frac{1}{2\lambda_{\max}}\|\tilde{\Phi}\|^2 \le \frac{1}{2}\tilde{\Phi}^T\Gamma^{-1}\tilde{\Phi} \le \frac{1}{2\lambda_{\min}}\|\tilde{\Phi}\|^2. \tag{109}$$

Combining Ineqs. (108) and (109), we have

$$0 \le \lambda_{1a}(V_1 + V_2 + \|\tilde{\Phi}\|^2) \le V_a \le \lambda_{2a}(V_1 + V_2 + \|\tilde{\Phi}\|^2), \quad (110)$$

where $\lambda_{1a} = \min\left(\lambda_1, \frac{1}{2\lambda_{\max}}\right)$ and $\lambda_{2a} = \max\left(\lambda_2, \frac{1}{2\lambda_{\min}}\right)$ are two positive constants. □

Appendix D. Proof of Lemma 7

Proof. We obtain the time derivation of the Lyapunov function candidate Eq. (49) as

$$\dot{V}_a = \dot{V} + \tilde{\Phi}^T \Gamma^{-1} \dot{\tilde{\Phi}}. \quad (111)$$

Substituting Eq. (47) into the second term of the Eq. (71), we have

$$\begin{aligned} \dot{V}_2 &= M_s u_a \dot{u}_a \\ &= -k u_a^2 + \mathrm{d} u_a - \mathrm{sgn}(u_a) \bar{\mathrm{d}} u_a + P\tilde{\Phi} u_a \\ &\le -k u_a^2 + P\tilde{\Phi} u_a. \end{aligned} \quad (112)$$

Applying the results of Lemma 5 and utilizing Ineqs. (81), (112) and (96) in $\dot{V}$, we obtain

$$\dot{V} \le -\lambda_3(V_1 + V_2) + P\tilde{\Phi} u_a + \varepsilon, \quad (113)$$

where λ_3 is defined in Eq. (106) and ε is defined in Appendix B. Application of Ineq. (113) into Eq. (111) yields

$$\dot{V}_a \le -\lambda_3(V_1 + V_2) + \tilde{\Phi}^T \left(P^T u_a + \Gamma^{-1} \dot{\tilde{\Phi}}\right) + \varepsilon. \quad (114)$$

Substituting Eq. (48) into Ineq. (114), we have

$$\begin{aligned} \dot{V}_a &\le -\lambda_3(V_1 + V_2) + r\tilde{\Phi}^T \hat{\Phi} + \varepsilon \\ &\le -\lambda_3(V_1 + V_2) - \frac{r}{2}\|\tilde{\Phi}\|^2 + \frac{r}{2}\|\Phi\|^2 + \varepsilon \\ &\le -\lambda_{3a}(V_1 + V_2 + \|\tilde{\Phi}\|^2) + \frac{r}{2}\|\Phi\|^2 + \varepsilon, \end{aligned} \quad (115)$$

where $\lambda_{3a} = \min\left(\lambda_3, \frac{r}{2}\right)$ is a positive constant. From Ineqs. (110) and (115), we have

$$\dot{V}_a \le -\lambda_a V_a + \psi, \quad (116)$$

where $\lambda_a = \lambda_{3a}/\lambda_{2a}$ and $\psi = \frac{r}{2}\|\Phi\|^2 + \varepsilon > 0$. □

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